Dynamic electrorheological effects and interparticle force between a pair of rotating spheres

Jones T. K. Wan,[1] K. W. Yu,[1] and G. Q. Gu[1,2]
[1]Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China
[2]College of Computer Engineering, University of Shanghai for Science and Technology, Shanghai 200 093, China
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We consider a two-particle system in which a particle is held fixed, and the other one rotates around the axis perpendicular to the line joining the particles' centers. The rotating particle leads to a displacement of its polarization charge on the surface. Our results show that the rotational motion of the particles generally reduces the force between the particles. The dependence of interparticle force on the angular velocity of rotation will be discussed.

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I. INTRODUCTION

The prediction of the strength of the electrorheological (ER) effect is the main concern in the theoretical investigation of ER fluids [1–5]. An ER fluid is a suspension of particles with high dielectric constant (conductivity) in a host medium of low dielectric constant (conductivity). The ER effect originates from the induced interaction between the polarized particles in an ER fluid. Upon the application of an external electric field, polarization charges are induced on the particle surfaces, leading to anisotropic forces between the particles. Under the influence of the ER effect, the particles in an ER fluid initially aggregate rapidly into chains within milliseconds and then the chains aggregate into columns within seconds [3,4]. The rapid field-induced transition between the fluid and solid phase makes this material both important for wide industrial applications such as shock absorbers, dampers and clutches, as well as for experimental and theoretical investigations.

In deriving the induced forces between the particles, existing theories assume that the particles are at rest [6–10]. In a realistic situation, the fluid flow exerts force and torque on the particles, setting the particles in both translational and rotational motions. For instance, the shear flow in an ER suspension exerts a torque on the particles, which leads to the rotation of particles about their centers [11]. Recent experiments showed that the induced forces between the rotating particles are markedly different from the values predicted by the existing theories that have not taken the motion of particles into account [12].

Wang designed a delicate experiment, in which a spherical particle is held fixed and the other one rotates uniformly, to investigate the dynamic ER effects [13]. The experimental result is that the force exerted on the rotating particle is smaller than that on the same particle at rest. In order to explain this effect, we formulate a model, which describes the relaxation process of the polarized charge on the surfaces of the rotating particles. We will compute the force between the polarized particles by the multiple image method [14]. We will show that the induced force is inversely proportional to the square of the product of the relaxation time and the angular velocity of the rotating particle.

II. DIPOLE MOMENT OF A ROTATING DIELECTRIC SPHERE

In this section, we consider two spherical particles, one of which is held fixed, and the other one rotates around the axis perpendicular to the line joining the particles centers (Fig. 1). Both particles are dielectric spheres of dielectric constant ϵ_1, suspended in a host medium of dielectric constant ϵ_m. The rotating (fixed) sphere has a radius $a(b)$. In this section, we neglect the mutual polarization between the particles. This assumption is valid if the distance between the particles is large. Upon the application of an electric field, polarization charges are induced on the surface of the spheres and there are induced dipole moments. Suppose the applied field is given by $\vec{E}_0=E_0\hat{z}$, the induced dipole moment is given by $\vec{p}_{a0}=p_{a0}\hat{z}$, where

$$p_{a0}=\epsilon_m E_0\left(\frac{\epsilon_1-\epsilon_m}{\epsilon_1+2\epsilon_m}\right)a^3, \tag{1}$$

and a similar expression is given for the fixed sphere. In the

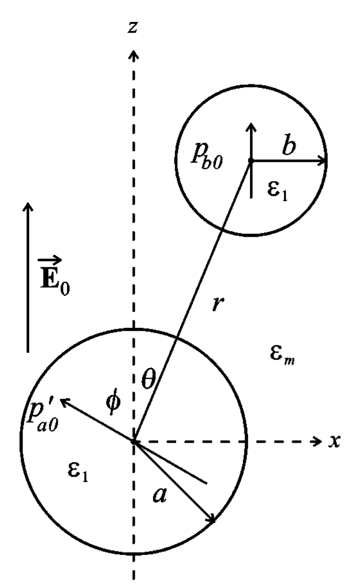

FIG. 1. A rotating polarized dielectric sphere interacting with another nonrotating polarized dielectric sphere.

presence of a rotational motion, a displacement of polarization charge occurs on the particle surface.

Suppose the sphere is undergoing an anticlockwise rotation with an angular velocity $\vec{\omega} = -\omega\hat{y}$. The induced dipole moment will rotate with the same angular velocity, and the rate of change of the induced dipole moment is given by

$$\frac{d\vec{p}}{dt} = \vec{\omega}\times\vec{p}. \tag{2}$$

However, the induced dipole moment can relax back to its original orientation (along $\vec{E}_0$) and magnitude. If the relaxation process is characterized by a relaxation time τ_r, we have

$$\frac{d\vec{p}}{dt} = -\frac{1}{\tau_r}(\vec{p}-\vec{p}_{a0}). \tag{3}$$

When a steady state is reached, we have

$$\vec{\omega}\times\vec{p} - \frac{1}{\tau_r}(\vec{p}-\vec{p}_{a0}) = 0. \tag{4}$$

Solving for the above equation, we obtain

$$\vec{p} = \frac{\vec{p}_{a0} + \tau_r(\vec{\omega}\times\vec{p}_{a0})}{1+(\omega\tau_r)^2}. \tag{5}$$

The result of Eq. (5) has a simple geometric interpretation. It can be shown that the angle between $\vec{p}_{a0}$ and $\vec{p}$ is equal to ϕ and the magnitude of $\vec{p}$ is given by $|\vec{p}| = p_{a0}\cos\phi$, where ϕ is given by $\tan\phi = \omega\tau_r$.

III. MULTIPLE IMAGE DIPOLES FOR A PAIR OF DIELECTRIC SPHERES

In this section, we generalize the multiple image method [14] to calculate the force between the polarized spheres, when the induced dipole moments are not parallel. Consider two dielectric spheres, the radii of which are given by a and b, respectively, and the separation between their center is r. Suppose both of the spheres are polarized with induced dipole moments $\vec{p}_{a0}$ and $\vec{p}_{b0}$, respectively. Due to mutual polarization effect, the dipole moment of each sphere will induce an image dipole on the other sphere. The induced image dipole will further induce another image dipole on the original sphere and hence multiple images are formed [14]. To express the image dipole moment, it is convenient to separate the original dipole moments into x (perpendicular to $\vec{E}_0$) and z components (parallel to $\vec{E}_0$):

$$\vec{p}_{a0} = p_{a0x}\hat{x} + p_{a0z}\hat{z}, \quad \vec{p}_{b0} = p_{b0x}\hat{x} + p_{b0z}\hat{z}. \tag{6}$$

The total dipole moments are given by

$$\vec{p}_a = p_{ax}\hat{x} + p_{az}\hat{z}, \quad \vec{p}_b = p_{bx}\hat{x} + p_{bz}\hat{z}. \tag{7}$$

The x components can be found by using the multiple image method [14]:

$$p_{ax} = \sum_{n=1}^{\infty}\left[\frac{p_{a0x}(b\sinh\alpha)^3(-\tau)^{2n-2}}{(b\sinh n\alpha + a\sinh(n-1)\alpha)^3} + \frac{p_{b0x}(a\sinh\alpha)^3(-\tau)^{2n-1}}{(r\sinh n\alpha)^3}\right], \tag{8}$$

$$p_{bx} = \sum_{n=1}^{\infty}\left[\frac{p_{b0x}(a\sinh\alpha)^3(-\tau)^{2n-2}}{(a\sinh n\alpha + b\sinh(n-1)\alpha)^3} + \frac{p_{a0x}(b\sinh\alpha)^3(-\tau)^{2n-1}}{(r\sinh n\alpha)^3}\right]. \tag{9}$$

The z components can be found similarly [14]:

$$p_{az} = \sum_{n=1}^{\infty}\left[\frac{p_{a0z}(b\sinh\alpha)^3(2\tau)^{2n-2}}{(b\sinh n\alpha + a\sinh(n-1)\alpha)^3} + \frac{p_{b0z}(a\sinh\alpha)^3(2\tau)^{2n-1}}{(r\sinh n\alpha)^3}\right], \tag{10}$$

$$p_{bz} = \sum_{n=1}^{\infty}\left[\frac{p_{b0z}(a\sinh\alpha)^3(2\tau)^{2n-2}}{(a\sinh n\alpha + b\sinh(n-1)\alpha)^3} + \frac{p_{a0z}(b\sinh\alpha)^3(2\tau)^{2n-1}}{(r\sinh n\alpha)^3}\right]. \tag{11}$$

The factor $(-\tau)$ and (2τ) apply to the transverse and longitudinal image dipole moments, respectively. The factor τ is known as the dielectric contrast and is given by

$$\tau = \frac{\epsilon_1 - \epsilon_m}{\epsilon_1 + \epsilon_m}. \tag{12}$$

The parameter α is given by

$$\cosh\alpha = \frac{r^2 - a^2 - b^2}{2ab}. \tag{13}$$

We should remark that the present generalization is an approximation only, because there is a more complicated image method for a dielectric sphere [15]. However, in the limit $\tau\to 1$, the above expressions reduce to the results of perfectly conducting spheres. We expect that this approximation will be good at high contrast $(\tau\to 1)$. We have checked the validity of the approximation by comparing the above analytic expressions with the numerical solution of an integral equation method [16]. The force between the spheres is given by the following expression, based on the energy considerations [17,18]:

$$\vec{F} = \frac{1}{2}\nabla[\vec{E}_0\cdot(\vec{p}_a + \vec{p}_b)] = F_r\hat{r} + F_\theta\hat{\theta}. \tag{14}$$

It should be remarked that $\vec{F}(\theta) = \vec{F}(\theta+\pi)$ by examining Fig. 1. We will calculate the results for an anticlockwise rotation $(\vec{\omega} = -\omega\hat{y})$ only. The results for a clockwise rotation can be found similarly.

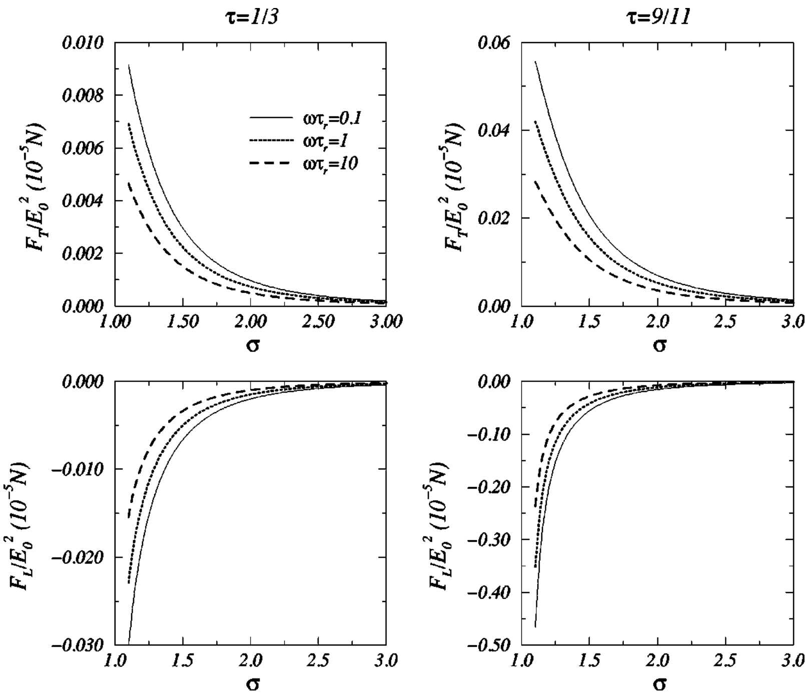

FIG. 2. Interparticle force F_T and F_L plotted against σ. The subscript T denotes the transverse dipole ($\theta=\pi/2$) and L the longitudinal dipole ($\theta=0$). The rotational motion of the particle generally reduces the interparticle force.

IV. NUMERICAL RESULTS

In this section, we report the separation and angular dependence of the force between the spheres. We will calculate the radial component (F_r) and the tangential component (F_θ) of the force and discuss their dependence on $\omega\tau_r$. Moreover, we let $b=a$ for convenience.

In Fig. 2, we plot the radial force between the spheres against the separation parameter σ, defined by $\sigma=r/(a+b)$. Two cases are considered, namely, the transverse and longitudinal field cases. In the transverse (longitudinal) field case, the line joining the spheres is perpendicular (parallel) to the applied field. We first examine the transverse field case, with $\omega\tau_r$ being chosen to be 0.1, 1, and 10, respectively. The dielectric contrast is chosen to be $\tau=1/3$ (low contrast) and $\tau=9/11$ (high contrast). From Fig. 2, our results show that the rotational motion of the particle generally reduces the force between the particles. In order to see the effects more clearly, we plot in Fig. 3 the products $F_T\sigma^4$ and $F_L\sigma^4$ against σ. It can be seen that for a large separation ($\sigma>3$), these quantities tend to be constant, indicating that the force varies as σ^{-4}. For a small separation ($\sigma<1.5$), the magnitude of the transverse force increases rapidly. The longitudinal field case shows a similar behavior: the rotational motion generally reduces the magnitude of the interparticle force.

We next discuss the angular dependence of the interparticle force (Fig. 4). We concentrate on the radial component

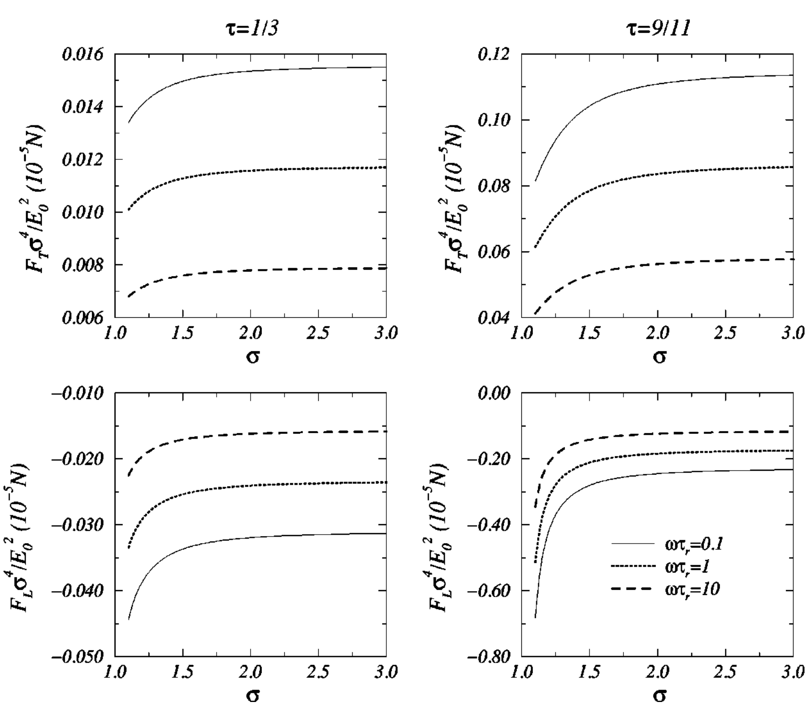

FIG. 3. Similar to Fig. 2, but for $F_T\sigma^4$ and $F_L\sigma^4$. The magnitude of the forces increases rapidly when the separation becomes small.

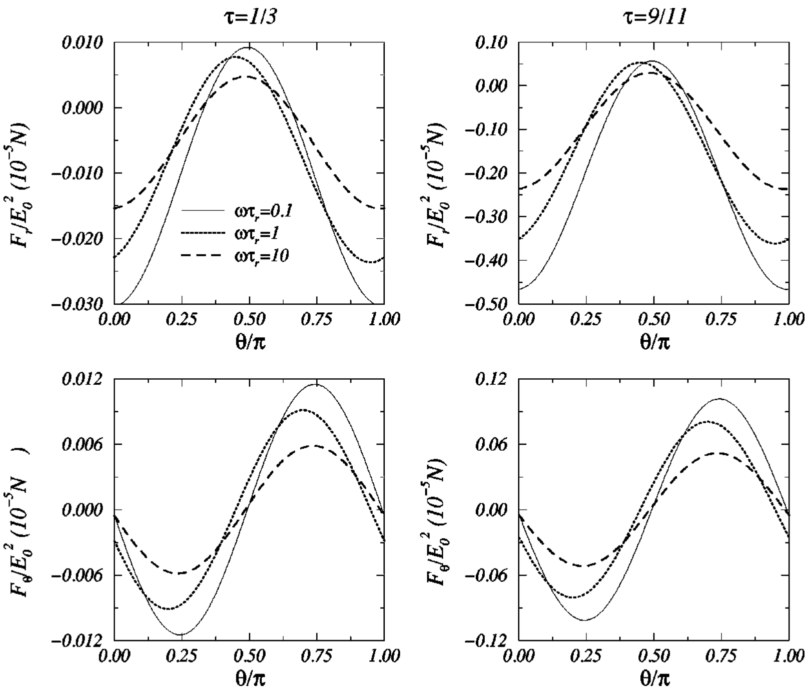

FIG. 4. The angular dependence of force. The angle is expressed in units of π, and the separation parameter is taken to be $\sigma=1.1$. The peaks shift to the direction of smaller θ. For $\omega\tau_r=1$, the angular locations $\theta=0$ and $\theta=\pi/2$ are no longer the equilibrium position of the tangential force.

(F_r) first. We can see that, for a small angular velocity ($\omega\tau_r=0.1$), the radial component attains its minimum and maximum value at $\theta\approx 0$ and $\theta\approx\pi/2$ respectively, which correspond to the longitudinal and transverse field case (Fig. 2). For the tangential force, $F_\theta\approx 0$ at $\theta=0$ and $\theta=\pi/2$. This is expected because when the angular velocity of the particle is small, the effect of rotation can be neglected. However, when the sphere rotates faster ($\omega\tau_r=1$), the peak shifts toward the direction of smaller θ. A similar conclusion can be drawn for the tangential force. In the lower panels of Fig. 4, the tangential force no longer vanishes at $\theta=0$ and $\theta=\pi/2$ for $\omega\tau_r=1$. From the above results, we conclude that the nonrotating particle is forced to move in the direction of increasing θ. More precisely, if originally the two particles are at $\theta=\pi/2$ while there is no rotational motion, the particles will repel each other and there is no tangential motion. However, if one of the particle begins to rotate, a tangential force will act on the other particle, forcing it to move downward, since $F_\theta>0$ at $\theta=\pi/2$.

In the opposite limit, when the angular velocity is large ($\omega\tau_r=10$), the peak of the radial force will return to the original position ($\theta=0$ and $\theta=\pi/2$), and $F_\theta=0$ when $\theta=0$ or $\theta=\pi/2$. Furthermore, the magnitude of the force is reduced to nearly one half of its original value (Fig. 5). It is because when the angular velocity is large, from $p'_{a0}=p_{a0}\cos\phi$ and $\tan\phi=\omega\tau_r$ (Fig. 1), the magnitude of the

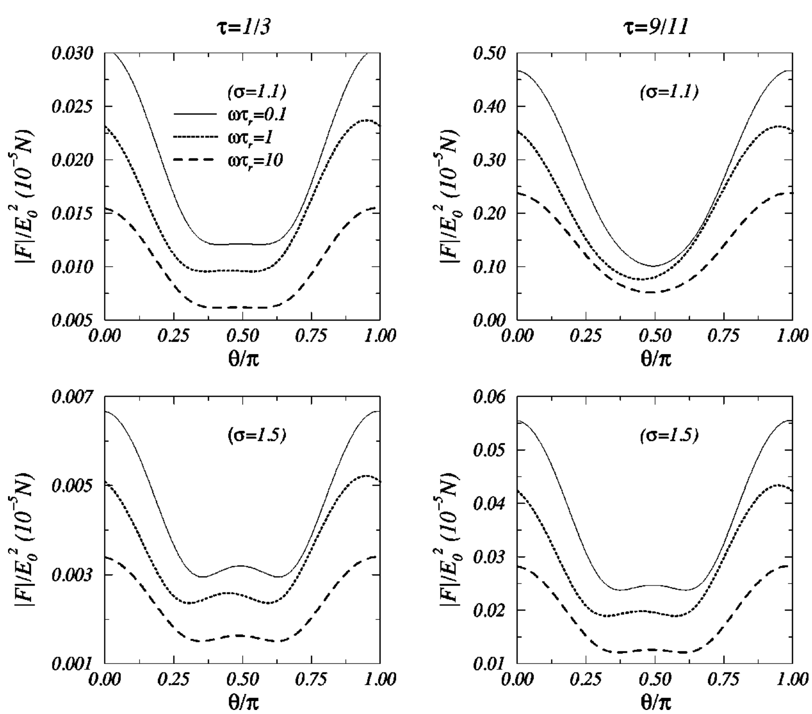

FIG. 5. The angular dependence of the magnitude of the force. The magnitude of the force is reduced to nearly one half of the original value.

steady-state dipole moment will be small. The situation is reminiscent of a polarized sphere interacting with a dielectric sphere with a vanishing polarization charge.

V. DISCUSSION AND CONCLUSION

Here a few comments on our results are in order. In this work, only one of the particles has been assumed to rotate. In a realistic situation, all the particles can rotate. As our theory is general, it can readily be applied to the case when all the particles are in rotational motion.

So far, our proposed relaxation time has no microscopic origin. If the relaxation process is originated from a finite conductivity of the particle or host medium, then we can estimate the relaxation time based on the Maxwell-Wagner theory of leaky dielectrics [19]. For a (nonrotating) spherical inclusion embedded in a host medium, the expression is

$$\tau_r = (\epsilon_1 + 2\epsilon_m)/(\sigma_1 + 2\sigma_m),$$

where σ_1, σ_m denote the conductivity of the sphere and host medium, respectively.

We may extend the Maxwell-Wagner theory to polarization relaxation of rotating particles. In this case, we should add a term $\rho_P \vec{v}$ to the polarization current density, where ρ_P is the polarization charge density and $\vec{v} = \vec{\omega} \times \vec{r}$ is the rotating velocity. However, it is not possible to convert the extra term into a dielectric constant and the generalization becomes more complicated due to the nonuniform polarization charge density inside the rotating spherical inclusions. We are currently examining the solution of the more complicated boundary-value problem.

However, we can make a crude estimate of the relaxation time. According to the geometry of the rotating sphere, the relaxation of polarization charge can be along the particle-fluid interface, so

$$1/\tau_r = \sigma_1/\epsilon_1 + \sigma_m/\epsilon_m .$$

When ϵ_1 is large compared with ϵ_m and $\sigma_m = 0$ (insulating host medium), both the expressions for τ_r reduce to the same result.

In conclusion, we have developed a model to calculate the interparticle force between a pair of dielectric spheres, in which one sphere is held fixed and the other is in rotational motion. We have calculated the force between the spheres by using the multiple image method. We have found that the effect of rotation is large when the angular velocity of the sphere is large ($\omega\tau_r \gg 1$). To investigate the dynamic ER effects, we have solved a differential equation that describes the dynamic polarization of the rotating particles. The theoretical problem described here can be realized in experiments [13].

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